Engineering Notes

Subsonic Induced Drag

KARL L. SANDERS*
Ryan Aeronautical Company, San Diego, Calif.

Nomenclature

ϵ_T = twist angle at wing tip (deg)
η = $2y/b$, dimensionless spanwise coordinate
λ = taper ratio (tip chord/root chord)
A = aspect ratio
$C_{L\alpha\infty}$ = section lift coefficient slope (1/deg)
e = span efficiency factor
C_L = total wing lift coefficient

Equations for Twist Distribution and Induced Drag

IN Ref. 1, pp. 4.1.5.2-2 and 4.1.5.2-3, Eq. (4.1.5.2-f) is given for the estimation of induced drag of cambered and/or twisted wings. This equation has been taken from Ref. 2; it is valid for a linear variation of the twist angle with the span and has been derived from a simple lifting-line method. Consequently, if accurate results are desired, the methods of Refs. 1 and 2 are confined to 1) a linear twist angle variation and 2) unswept wings of aspect ratios equal to or beyond 5 approximately. For wings of any planform and any twist distribution, the method of Ref. 3 [p. 5, Eq. (9)] is suggested.

For the special case of unswept wings of aspect ratios equal to or beyond 4, having a special twist distribution that maintains linear leading and trailing edges, the method given in Ref. 4 is suggested. Linear leading and trailing edges are advantageous in wing design and manufacture and should therefore be approached whenever possible. The induced drag equation of Ref. 4 is given below and based on Multhopp's lifting-line method.[6]

For trapezoidal planforms with linear leading and trailing edges, the twist angle at any spanwise station is given by

$$\epsilon(\eta) = \epsilon_T \eta \lambda / [1 - \eta(1 - \lambda)] \qquad (1)$$

Note that only for $\lambda = 1.0$ the twist angle variation becomes linear.

The induced drag coefficient for trapezoidal planforms, or those nearly fitting that type, using foregoing twist function, is

$$C_{D_i} = C_L^2/\pi A e + K_0 \epsilon_T^2 C_{L\alpha\,\infty}^2 + K \epsilon_T C_{L\alpha\infty} C_L \qquad (2)$$

$$\text{for } A \geq 5 \text{ and } \lambda \leq 1.0$$

The constants K_0 and K were obtained from Multhopp lift distributions[6] for aspect ratios and taper ratios ranging from 4 to 20 and 0.2 to 1.0, respectively. In order to expedite interpolations, K_0 and K were curve-fitted and given in Ref. 5 as

$$K_0 = (0.0088\lambda - 0.0051\lambda^2)(1 - 0.0006A^2) \qquad (3)$$

$$K = 0.0134(\lambda - 0.3) - 0.0037\lambda^2 \qquad (4)$$

The factors K_0 and K are valid for a steadily increasing positive or negative twist from the plane of symmetry toward the tips. However, they are also valid in good approximation for small deviations herefrom, for instance, if the twist starts at some small distance outboard from the plane of symmetry.

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* Design Specialist. Member AIAA.

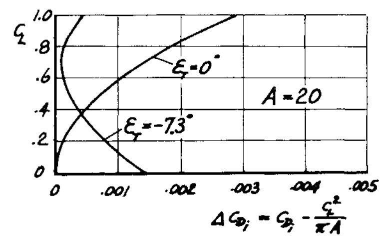

Fig. 1 Twist induced drag comparison for a −7.3° twisted and untwisted rectangular wing of aspect ratio 20. Note that, for the higher lift coefficients, the untwisted wing has higher induced drag than the twisted one.

On the other hand, if the twist discontinues at a small distance from the wing tip, it is more accurate to extrapolate the twist angle toward the tip and to use the tip value as "nominal."

The factor K_0 is positive. Therefore any twist of the trapezoidal wing, even at $C_L = 0$, always causes an increment in induced drag. Older K_0 values from Ref. 2, based on linear twist, are approximately 30 to 50% higher. The factor K may have positive as well as negative values. This contribution of the twist induced drag may therefore increase or decrease the induced drag, depending on the combination of the normally negative twist angle ϵ_T with a positive or negative value of K. K becomes positive if $\lambda > 0.35$, that is, if the taper ratio of the trapezoidal wing remains within the usual range. In general then, this second contribution for the twist induced drag will be negative and will reduce the total induced drag.

Induced Drag Comparisons

This leads immediately to the fact that the total induced drag of a negatively twisted trapezoidal wing at high lift coefficients results smaller than the drag of the same but untwisted wing. For example, a rectangular wing of $A = 20$ and $\epsilon_T = -7.3°$ at a $C_L = 0.4$ is already better than the corresponding untwisted wing, as is shown in Fig. 1.

The explanation for the lesser induced drag of the twisted rectangular wing lies in the lift distribution that approaches more closely the ideal elliptical shape. It is known that all wings of nonelliptical loading can be brought to a minimum drag by a special twist, which is not necessarily of the type governed by Eq. (1). The foregoing example of the rectangular wing ($\lambda = 1.0$), however, was carried out using the resulting linear, negative twist.

Discontinuous Twist Distribution

From time to time it may be required to consider certain types of twist, the maximum value of which does not occur at the wing tip, but farther inboard, for instance, at midsemispan. Such a twist reversal (or constancy) as compared to the continuous twist governed by Eq. (1) is shown in Fig. 2. With respect to avoiding the onset of stall, both twist shapes may in certain cases do the same job. However, with respect to drag they are quite different from each other, as is shown

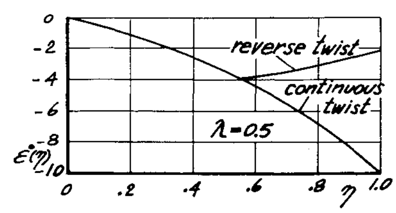

Fig. 2 A continuous and a reversed twist distribution.

in Fig. 3, where the twist induced drag of two wings of aspect ratio 20 having different twists are plotted. As compared to the continuous twist, the reversed twist is by far superior at the lower lift coefficients, that is, at higher speeds. For lower aspect ratios these differences become less pronounced.

The additional induced drag of such special twist types cannot, however, be satisfactorily calculated by the K factors given in this note, and instead, requires a special computation,

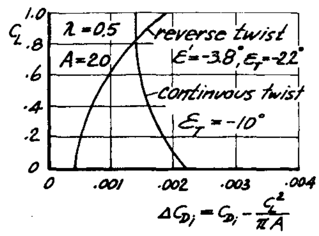

Fig. 3 Comparison of twist induced drag of a continuous and a reversed twist distribution for a trapezoidal wing.

using, e.g., Ref. 3. The outboard twist function must, of course, be optimized, such as to yield the desired drag characteristics. The resulting discontinuities in the wing skin are preferably buried in a nacelle, or if not available, locally faired.

Summary

Wing twist is primarily applied for two reasons: 1) to prevent the local onset of stall over some portion of the wing and 2) to achieve a minimum of induced drag at some optimum lift coefficient for cruise flight. Inspection of Eq. (2) reveals that induced drag consists of the following three interacting elements: lift, planform and section shape, and twist.

It was pointed out that the twist equation [Eq. (1)] has distinct advantages in wing design and manufacture, owing to the resulting linear leading and trailing edges. Twist induced drag coefficients determined by Eqs. (1) and (2), as compared to those of Ref. 2 (linear twist), are of lesser magnitude. Locally reversed or constant twist offers opportunities to minimize the twist induced drag for certain values of lift coefficient (i.e., speed), as shown in Fig. 3. The corresponding optimum twist distribution and induced drag can be found only by special computations. It should be noted that the differences in induced drag between wings of zero, continuous, and/or reversed twist become less pronounced as aspect ratios decrease.

The span efficiency "e" factor in the first term of Eq. (2) should be estimated preferably by the method of Ref. 7 which accounts for effects of Reynolds number. Wing theory, on the other hand, yields the well-known Glauert correction factor (Ref. 6, pp. 205, 215). It is suggested that the method given in Ref. 1 be replaced by that of Ref. 3 and this note.

References

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Multitapered Wings

B. SAELMAN*
Lockheed Aircraft Corporation, Burbank, Calif.

Nomenclature

W_1 = weight of outer panel, lb
W_2 = weight of inner panel, lb
W = total weight, lb
W_g = design gross weight, lb
N = load factor
ρ = material density, lb/in.^{3}
p = pressure, psi
M_1 = bending moment at station y in outer panel, in.-lb
M_2 = bending moment at station y in inner panel, in.-lb
h = maximum depth of section at station y, in.
k = ratio of effective depth to maximum depth
h_B = maximum depth at break station, in.
h_T = maximum depth at tip, in.
h_R = maximum depth at root, in.
c_R = root chord, in.
C_T = tip chord, in.
c_B = chord at break, in.
λ_1 = taper ratio of outer panel
λ_2 = taper ratio of inner panel
$f(\lambda)$ = taper ratio function $= \frac{1}{3} + [\lambda/(1-\lambda)] - 2[\lambda/(1-\lambda)]^2 + 2[\lambda/(1-\lambda)]^3 \log(1/\lambda)$
S_1 = area of outer panel, in.^{2}
S_2 = area of inner panel, in.^{2}
AR_1 = aspect ratio of outer panel
AR_2 = aspect ratio of inner panel
S = total wing area, in.^{2}
Λ = sweep of the 50% chord, deg
W_{sec} = weight of secondary structure, lb
ϕ = $(c_B - C_T)/e$
ψ = $c_R - C_B/a$
f = average working stress, psi

In most theoretical analyses of wings it is assumed that the wing has constant planform taper. However, there are many examples in the design of aircraft in which it has been found advantageous to employ a wing composed of sections of different rates of taper. In Ref. 1 a theoretical wing weight was obtained for wings of constant planform taper and of constant thickness ratio. An expression for the weight of a wing consisting of two sections of different planform taper ratios is developed in this paper (Fig. 1), and it is clear that the method can be extended to obtain the weights of wings of more than two tapers. Considering the outer panel and assuming a uniform pressure distribution, the bending moment

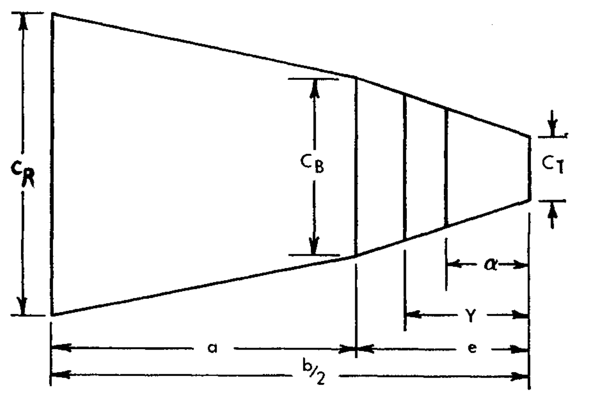

Fig. 1 Weight of wing consisting of two sections of different planform tapers.

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* Design Specialist.